Multicriteria Analysis

Gianfranco Atzeni

Laboratorio di valutazione delle decisioni di investimento

Introduction

In the investment decision very often we are asked to compare alternative investment projects involving different inputs and yielding different results/outputs.

Other situations require the comparisons among alternative projects (e.g. a new product) with different characteristics or attributes, or different level of these attributes.

The most common form of analysis in government is cost-effectiveness analysis (CEA), where the costs of alternative ways of providing similar kinds of output are compared.

A widely used methodology is also cost benefit analysis (CBA), in which some non-marketed outputs are explicitly valued in money terms.

Introduction

CEA and CBA are analytical ways of comparing different forms of input or output:

- in these cases by giving them money values
- might themselves be regarded as examples of multi-criteria analysis (MCA)

MCA is indeed sometimes referred to as multi-attribute analysis and the words criterion and attribute are often used synonymously:

- Attribute is also sometimes used to refer to a measurable criterion;
- we will use the words attribute, characteristic, criterion as synonyms.

The decision making process

Decision making about proposals for future action should normally follow the sequence below.

The following process might apply to the development of a policy, a program or a project:

- Identifying objectives
- Identifying options for achieving the objectives
- dentifying the criteria to be used to compare the options
- Analysis of the options
- Making choices, and
- Feedback

Sequences of decisions through time

Decision making often involves the use of decision trees to help identify good strategies for planning a response to a set of interdependent decisions sequenced through time.

The actual outcome of each of the individual decisions at each stage is not known with certainty.

Appropriate analysis of the tree allows the decision maker to develop, from the outset of the decision process, a **contingent decision** strategy.

Multi-criteria Analysis techniques

Many MCA approaches:

- All MCA approaches make the options and their contribution to the different criteria explicit.
- They differ in how they combine the data.
- Formal MCA techniques usually provide an explicit relative weighting system for the different criteria.

The main role of the techniques is to handle large amounts of complex information in a consistent way.

Limitations of MCA

Subjectivity. MCA puts emphasis on the judgement of the **decision making team**:

- in establishing objectives and criteria,
- estimating relative importance weights,
- in judging the contribution of each option to each performance criterion.

No welfare concerns. MCA cannot show that an action adds more to welfare than it detracts.

- The 'best' option can be inconsistent with improving welfare,
- so doing nothing could in principle be preferable.

The Performance Matrix

In the performance matrix below each column describes an option and each row describes the performance of the options against each criterion. Or, in other terms, the level of the attribute/characteristic belonging to a given option (vacuum cleaner).

The individual performance assessments are often numerical, but may also be expressed as ‘bullet point’ scores, or color coding.

Some of these criteria are measured in cardinal numbers (price, dust capacity, charging time...), one in binary terms (detachable handle, YES=1 and NO=0), and one in qualitative terms (Type).

View all

	 AEG **ergorapido AG3105**	 Bissell **MultiReach ION+ 1311 4**	 Black & Decker **FEJ520**	 Bosch **Athlet BCH65MGKGB/ 02**
Price	£149.99 (Today's best price) View retailer	£209.99 (Today's best price) Compare 2 retailers	£130.00 (Typical price)	£149.99 (Today's best price) Compare 4 retailers
Specification				
Charge time	2 hours 2 minutes	3 hours 2 minutes	4 hours 51 minutes	4 hours 0 minutes
Detachable hand-held	No	No	Yes	No
Dust capacity in litres	0.4	0.4	0.5	0.4
Run time	46	49	27	48
Turbo run time	15 minutes	24 minutes	12 minutes	16 minutes
Type	Cordless	Cordless	Cordless	Cordless
Warranty length	-	-	-	-
Weight	2.5 kilograms	2.8 kilograms	2.5 kilograms	3.4 kilograms

The Performance Matrix

In a basic form of MCA this performance matrix may be the final product of the analysis:

- The decision makers are then left with the task of assessing the extent to which their objectives are met by the entries in the matrix.
- In analytically more sophisticated MCA techniques the information in the basic matrix is usually converted into consistent numerical values.

Weighting and scoring

- **SCORING**: the expected consequences of each option are assigned a numerical score on a strength of preference scale for each option for each criterion. More preferred options score higher on the scale, and less preferred options score lower. In practice, scales extending from 0 to 100 are often used.
- **WEIGHTING**: numerical weights are assigned to define, for each criterion, the relative valuations of a shift between the top and bottom of the chosen scale.

Weighting and scoring

- These two components are combined using mathematical routines to give an overall assessment of each option being appraised.
- Individuals need to provide those inputs that they are best suited to provide.
- Computers handle the detailed information in a way that is consistent with the preferences that have been revealed by these human inputs.

These approaches are often referred to as **compensatory MCA** techniques, since low scores on one criterion may be compensated by high scores on another.

Mutual independence of preferences

- The most common way to combine scores on criteria, and relevant weights between criteria, is to calculate a simple weighted average of scores.
- This requires the **mutual independence of preferences**: the judged strength of preference for an option on one criterion/attribute/charactheristic will be independent of its judged strength of preference on another.

Dominated options

Dominance occurs when one option performs at least as well as another on all criteria and strictly better than the other on at least one criterion.

In principle, one option might dominate all others, but in practice this is unlikely.

When it does occur, it is helpful to ask if there is some advantage of the dominated option that is not represented by the criteria. (e.g. length of the power wire in our example)

Dominance analysis enable the decision-making team to eliminate dominated options from further consideration.

Trade offs

- It is important to determine whether trade-offs between different criteria are acceptable, so that good performance on one criterion can in principle compensate for weaker performance on another.
- Where compensation is acceptable, most MCA methods involve implicit or explicit aggregation of each option's performance across all the criteria to form an overall assessment of each option, on the basis of which the set of options can be compared.
- The principal difference between the main families of MCA methods is the way in which this aggregation is done.

Multi-attribute utility theory

- It derives from the work of von Neumann and Morgenstern (1947), and Savage (1954)
- Keeney and Raiffa (1976)developed a set of procedures, consistent with the earlier normative foundations, which would allow decision makers to evaluate multi-criteria options in practice.

There are three building blocks for their procedures:

1. the performance matrix;
2. the procedures to determine whether criteria are independent of each other or not;
3. the estimation of the parameters in a function which allow the estimation of a single number index, U (overall valuation),

Linear additive models

If criteria are **preferentially independent** of each other and without uncertainty, then the simple linear additive evaluation model is applicable. This show how option's values on the many criteria can be combined into one overall value:

- by multiplying the value score on each criterion by the weight of that criterion;
- then adding all those weighted scores together

Mutual independence of preferences

Can you assign preference scores for the options on one criterion without knowing what the options' preference scores are on any other criteria?

If the answer is yes, then this criterion is preference independent of the others.

The question then is asked for each of the remaining criteria in turn.

If the answer is always yes, then the criteria are considered to be **mutually preference independent**.

The Analytical Hierarchy Process

It also develops a linear additive model and uses procedures for deriving the weights and the scores achieved by alternatives which are based, respectively, on **pairwise comparisons** between criteria and between options.

- For example, in assessing weights, the decision maker is asked a series of questions, each of which asks how important one particular criterion is relative to another for the decision being addressed.

Limitation of AHP

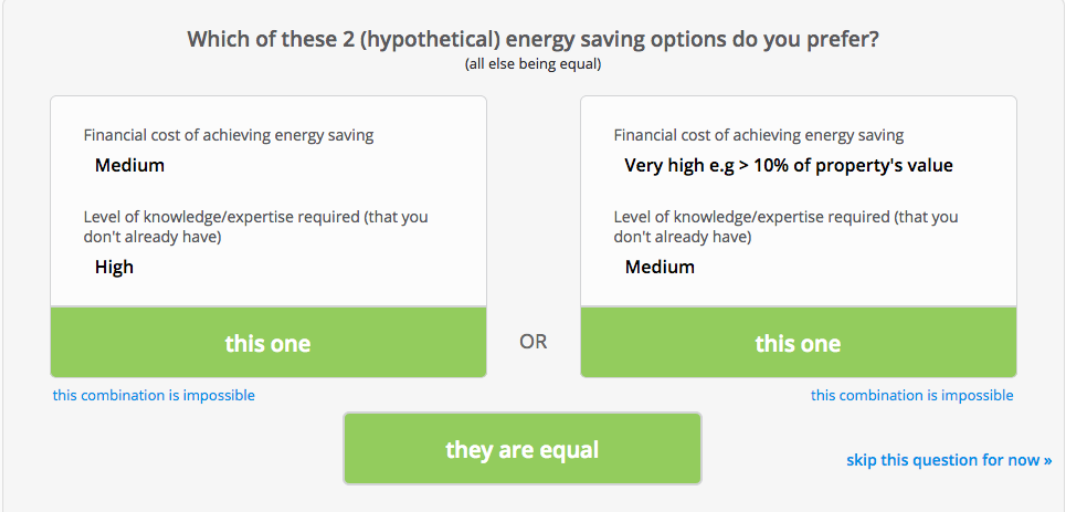
Doubts have been raised about the theoretical foundations of the AHP.

In particular, the **rank reversal** phenomenon has caused concern:

- this is the possibility that, simply by adding another option to the list of options being evaluated, the ranking of two other options, not related in any way to the new one, can be reversed.

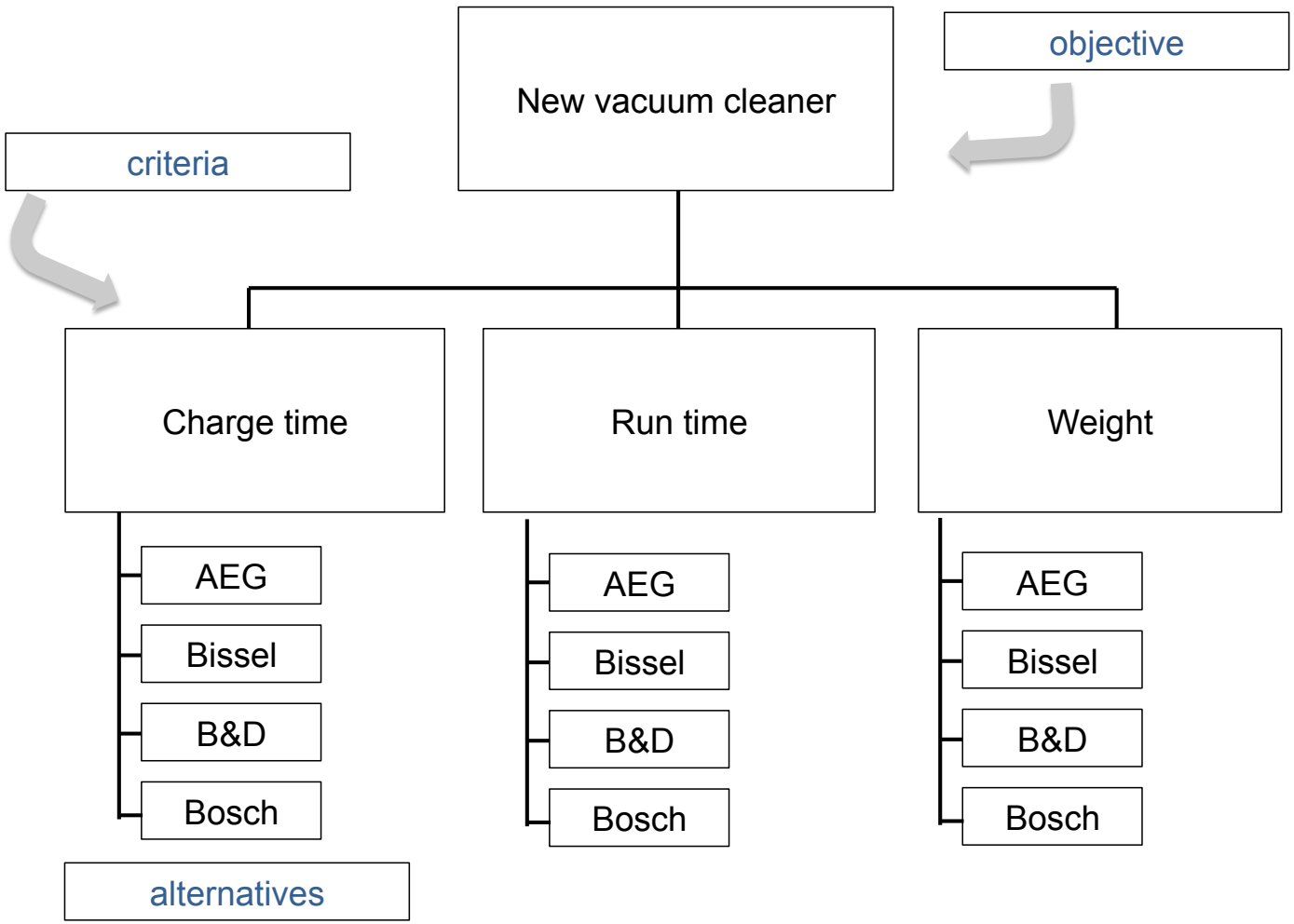
How AHP works

Many different approach to assign weights to criteria or attributes. For example PAPRIKA (**P**otentially **A**ll **P**airwise **R**an**K**ings of all possible **A**lternatives) ask question about two hypothetical projects. Then it assign scores to all alternatives.



How is it possible to obtain choices after the scores have been assigned?

Example of hierarchical tree



Parwise comparisons

Using pairwise comparisons the relative importance of one criterion over another can be expressed.

1= equal 3= moderate 5= strong 7=very strong 9=extreme

	charge time	run time	weight
charge time	1/1	1/2	3/1
run time		1/1	4/1
weight			1/1

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charge time	1/1	1/2	3/1
run time	2/1	1/1	4/1
weight	1/3	1/4	1/1

How to get priorities

How to get ranking priorities from a pairwise matrix?

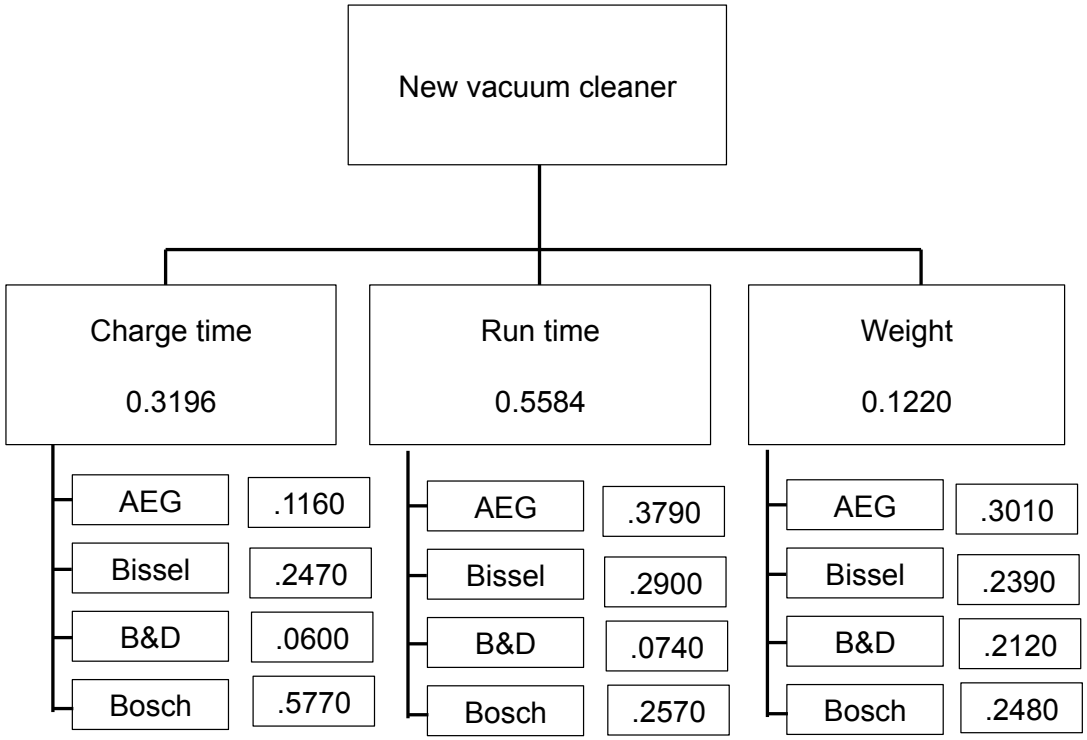
Saaty (1990) shows that the eigenvector is the best approach.

1. Raise the pairwise matrix to powers that are successively squared each time
2. the row sums are then calculated and normalized
3. the computer is instructed to stop when the difference between these sums in two consecutive calculations is smaller than a prescribed value

See spreadsheet file.

The same for the alternatives i.e. vacuum cleaners

Pairwise comparison of the four vacuum cleaners in term of each of the characteristics gives the eigenvector for each vacuum cleaner in term of charge time, run time, weight.



How to obtain the ranking vector

	charge time	run time	weight		criteria ranking	
AEG	.1160	.3790	.3010		.3196	char. time
Bissel	.2470	.2900	.2390	times	.5584	run time
B&D	.0600	.0740	.2120			
Bosch	.5570	.2570	.2480		.1220	weight

i.e for AEG $(.1160 \times .3196) + (.3790 \times .5584) + (.3010 \times .1220) = .3060$

AEG	.3060
Bissel	.2720
B&D	.0940
Bosch	**.3280**

The winner is Bosch! The Bosch is the highest ranked vacuum cleaner.

What about costs?

In the example above we ranked the vacuum cleaners giving a score that represents the benefit of each vacuum cleaner.

We did not take into consideration the price of the apparel.

In many cases when evaluating alternative investment projects is better to set aside costs until benefits are assessed.

A way to take into consideration costs is to compute **benefit to cost ratios**.

Benefit to cost ratio

	price/cost	normalized cost	benefit-cost ratio		
AEG	150	.2343	.3060/.2343	=	1.3060
Bissel	210	.3281	.2720/.3281	=	.8290
B&D	130	.2031	.0940/.2031	=	.4628
Bosch	150	.2343	.3280/.2343	=	**1.3999**
Total=	640	1.000			

Bosch vacuum cleaner is again the winner!

Work in groups

Go to www.1000minds.com

Choose one of the topic you find listed there:

1. Progetti di riqualificazione dell'ambiente retrodunale della spiaggia della Pelosa
2. Progetti di riqualificazione del centro storico di Sassari
3. Valutazione di diversi impianti di produzione di energia in Sardegna
4. Allocazione di prestiti di microcredito in Sardegna
5. Quale è la miglior squadra di basket di serie A

Prioritize at least 4 different projects. For the basket teams prioritize all the teams in Serie A (see www.legabasket.it).

Explain the choice of criteria/attributes and of levels and show your results in a max 10 slide presentation. Due date: 6th March 2017