Decision processes

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Introduction

In many situations decisions are taken in a sequence, following a path.

Consider for example the consumer choice to buy new trousers. The choice to buy a pair of blue Benetton trousers is not independent to the decision process that leads to it.

First I need to decide whether I need new trousers, then whether I want brand or non-brand trousers, then I evaluate the various alternatives among brand trousers, and also I have to decide whether I want pale or dark colors, and so on.

For many decisions we can build a decision tree in which each decision is nested in other previous one.

Independence of irrelevant alternatives

The fact that decisions are often nested in a tree structure has an important implication:

we cannot impose the restriction that the choice between any two pairs or alternative is simply binary (as in logit model).

This assumption, as we have seen for the Random Parameter model, is called **Independence of Irrelevant Alternatives** (**IIA**), which is too restrictive as can be illustrated in the famous ”red bus/blue bus” problem.

Red bus/blue bus problem

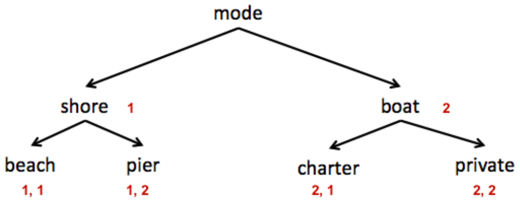
Suppose commute-mode alternatives are car, blu bus and red bus. Under the IIA assumption the probability of commuting by car, given that we have the option use the car or a red bus, is independent of whether we introduce the option of commuting by blue bus.

If the blue bus is the same as red bus except color, it has no or little impact on the probability to use a car. However, the introduction of the blue bus should halve use of red bus, increasing the conditional probability of car use given commute by car ore red bus.

Under the econometric point of view errors of blue and red bus are highly correlated.

Nesting structure

Consider the decision on fishing mode of the previous lectures. We can assume a distinction between shore and boat fishing, that results in the decision tree depicted below



The shore-boat is called **level 1** or limb. The next level is called **level 2** or branch, and so on in case of additional levels.

Correlated errors

The tree structure implies that the errors of the decisions in level 2 are correlated in each branch.

In our example $\varepsilon_{i,beach}, \varepsilon_{i,pier}$ are a bivariate correlated pair; and $\varepsilon_{i,private}, \varepsilon_{i,charter}$ are a bivariate correlated pair, and the two pairs are independent. Therefore, the Conditional logit model is a special case in which all errors are independent.

Random utility with nested structure

Denote alternatives by subscripts (j,k), where j denotes the limb (level 1) and k denotes the branch (level 2) within the limb. Different limbs can have different number of branches, including just one branch. For example (2,1) denotes the first alternative in the second limb (charter fish mode). The two level random utility is defined as follows:

$$U_{jk} + \varepsilon_{jk} = z_j'\alpha + x_{jk}'\beta_j + \varepsilon_{jk} \tag{1}$$

for $j = 1, ..., J$ and $k = 1, ..., K$. z_j varies over the limbs only and x_{jk} varies over both limbs and branches. In this case we are considering only alternative-specific regressors (no i subscripts).

Random utility with nested structure

The probability that alternative (j,k) is chosen is:

$$p_{ik} = p_j \times p_{k|j} = \frac{e^{(z_j'\alpha + \tau_j I_j)}}{\sum_{m=1}^{J} e^{(z_m'\alpha + \tau_m I_m)}} \times \frac{e^{(x_{jk}'\beta_j/\tau_j)}}{\sum_{t=1}^{K_j} e^{(x_{jl}'\beta_j/\tau_j)}} \quad (2)$$

where $I_j = ln(\sum_{t=1}^{K_j} e^{(x_{jl}'\beta_j/\tau_j)}$ and it is called the inclusive value.

The NL probabilities are the product of two CL probabilities p_j and $p_{k|j}$.

τ_j are the dissimilarities parameters which correspond to the degree of dissimilarity between the alternatives within one nest. The model is consistent with RUM if all τ lie in the unit interval.

Estimation: define the nesting structure

We need to define the nesting structure by creating a variable `type` called `shore` for the `pier` and `beach` alternatives, and it is called `boat` for the `charter` and `private` alternatives.

```
. nlogitgen type=fishmode(shore: pier|beach, boat: private | charter)
new variable type is generated with 2 groups
label list lb_type
lb_type:
           1 shore
           2 boat
```

This gives the following tree that can be checked using the `nlogittree` command.

```
tree structure specified for the nested logit model

 type    N        fishmode  N     k
-----------------------------------
 shore 2364 ─┬─ beach     1182  134
             └─ pier      1182  178
 boat  2364 ─┬─ charter   1182  452
             └─ private   1182  418
-----------------------------------
                   total  4728 1182

k = number of times alternative is chosen
N = number of observations at each level
```

NL estimation

For the estimation we use the command `nlogit` in which we need to define level-1 equation for type, without any regressors. Then we define the level-2 equations that depends on `income` and intercept.

Results

```
RUM-consistent nested logit regression         Number of obs      =       4728
Case variable: id                              Number of cases    =       1182

Alternative variable: fishmode                 Alts per case: min =          4
                                                              avg =        4.0
                                                              max =          4

                                                  Wald chi2(5)    =     212.65
Log likelihood = -1192.4116                       Prob > chi2     =     0.0000
```

d	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
fishmode						
p	-.0267478	.0018715	-14.29	0.000	-.030416	-.0230797
q	1.346902	.2827218	4.76	0.000	.7927779	1.901027
fishmode equations						
beach						
income	0	(base)				
_cons	0	(base)				
charter						
income	-5.465855	12.4429	-0.44	0.660	-29.8535	18.92179
_cons	48.37026	96.63506	0.50	0.617	-141.031	237.7715
pier						
income	-6.425322	12.85416	-0.50	0.617	-31.619	18.76836
_cons	39.88896	81.36795	0.49	0.624	-119.5893	199.3672
private						
income	-1.292133	2.046463	-0.63	0.528	-5.303126	2.71886
_cons	28.31695	57.03556	0.50	0.620	-83.4707	140.1046
dissimilarity parameters						
type						
/shore_tau	55.96862	118.9133			-177.0971	289.0344
/boat_tau	32.55049	86.28528			-136.5656	201.6665

Results

- the coefficient of the variable `p` is almost unchanged compare to the CL model, but all the other coefficients changed a lot.
- if the two dissimilarity parameters τ are both equal to 1 would mean that the NL model reduces to the CL (no need to define a nested structure for the alternatives)
- the `LR test for IIA (tau=1): chi2(2)=45.43 Prob > chi2= 0.0000` tells that we reject the CL model in favor of NL.
- the dissimilarity parameters are much greater than 1 $\rightarrow$ the fitted model is not consistent with Random Utility.

Predicted probabilitites

The `predict` command with the `pr` option provides predicted probabilities for level 1, level 2 and so on. In our example the level 2 probabilities refer to each of the four alternatives, which is want we want to know.

```
. predict plevel1 plevel2, pr

.
. tabulate fishmode, sum (plevel2)
```

fishmode	Summary of Pr(fishmode alternatives) Mean	Std. Dev.	Freq.
beach	.11319965	.13342355	1182
charter	.38065552	.15749307	1182
pier	.15075682	.16988656	1182
private	.35538801	.16426111	1182
Total	.25	.19693884	4728